Putting $x=\cos\theta+i\sin\theta$, and employing the formula $(\cos\theta+i\sin\theta)^m=\cos m\theta+i\sin m\theta$, we obtain after putting the real parts of both members equal, and making all necessary reductions, for the sum of the second series

$$=\frac{\cos\theta-n\cos n\theta+(n-1)\cos(n+1)\theta}{4\sin^2\frac{1}{2}\theta};$$

so that the sum of the given series

$$=\frac{n\cos\frac{1}{2}(n+1)\theta\sin\frac{1}{2}n\theta}{\sin\frac{1}{2}\theta}+\frac{\cos\theta-n\cos n\theta+(n-1)\cos(n+1)\theta}{4\sin^2\frac{1}{2}\theta}.$$

To test this formula we must of course, leave the coefficient n of the first expression unchanged, while in all the other factors and terms which involve n, n must be put successively$=1$, 2, 3, 4, etc.

Also solved by *E. W. MORRELL.*

69. Proposed by C. E. WHITE, A. M., Trafalgar, Indiana.

Prove that $x^n\pm x^{n-1}+x^{n-2}\pm\dots\dots+(\pm1)^{n-1}x+(\pm1)^n=(x\mp1)^n\pm A(x\mp1)^{n-1}+B(x\mp1)^{n-2}\pm\dots\dots+(\pm1)^nx$, where A, B, C, ... are the binomial coefficients of the $(n+1)$th order.

I. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

$$x^n\pm x^{n-1}+x^{n-2}+\dots\dots, \text{ etc.}=(x^{n+1}-1)/(x-1) \dots\dots\dots\dots(1),$$

or $x^{n+1}+1)/(x+1)$.............. ......(2), $=\{[(x-1)+1]^{n+1}-1\}/(x-1)$, or $\{[(x+1)-1]^{n+1}+1\}/(x+1)$, $=(x-1)^n+C^2_{n+1}(x-1)^{n-1}+C^3_{n+1}(x-1)^{n-2}+\dots\dots$,

$$\text{or } (x+1)^n-C^2_{n+1}(x+1)^{n-1}+C^3_{n+1}(x-1)^{n-2}-\dots\dots,$$

$$=(x\mp1)^n\pm A(x\mp1)^{n-1}+B(x\mp1)^{n-2}+\dots\dots$$

II. Solution by E. W. MORRELL, Professor of Mathematics in Montpelier Seminary, Montpelier, Vermont.

Let $K=x^n\pm x^{n-1}+x^{n-2}\pm\dots\dots+(\pm1)^{n-1}x+(\pm1)^n$.

Put $x=y\pm1$, expanding and observing that the sign of the last term of each expression is $\pm$ if n is odd but $+$ if n is even, we may write:

$$x^n=(y\pm1)^n=y^n\pm ny^{n-1}+\tfrac{1}{2}[n(n-2)]y^{n-2}\pm\dots\dots\dots\dots+(\pm1)^{n-1}ny+(\pm1)^n$$

$$\pm x^{n-1}=\pm(y\pm1)^{n-1}=\pm y^{n-1}+(n-1)y^{n-2}\pm\dots\dots\dots+(\pm1)^{n-1}(n-1)y+(\pm1)^n$$

$$x^{n-2}=(y\pm1)^{n-2}=\dots\dots\dots\dots y^{n-2}\pm\dots\dots\dots+(\pm1)^{n-1}(n-2)y+(\pm1)^n$$

etc ....etc.

$$(\pm1)^{n-1}x=(\pm1)^{n-1}(y\pm1)=\dots\dots\dots\dots\dots\dots\dots\dots\dots(\pm1)^{n-1}y+(\pm1)^{n}$$

$$(\pm1)^{n}=\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(\pm1)^{n}.$$

By adding, and simplifying the coefficients of y, we have

$$K=y^{n}\pm(n+1)y^{n-1}+\tfrac{1}{2}[(n+1)n]y^{n-2}\pm\dots+(\pm1)^{n-1}\tfrac{1}{2}[(n+1)n]y+(\pm1)^{n}(n+1),$$

which has binomial coefficients of the $(n+1)$th order. Substituting A, B, C, ... for the coefficients and restoring the values of y,

$$K=(x\mp1)^{n}\pm A(x\mp1)^{n-1}+B(x\mp1)^{n-2}\pm\dots\dots+(\pm1)^{n-1}B(x\mp1)+(\pm1)^{n}A.$$

[Expanding and combining the terms of the second member, we get the first member for a result. ZERR.]

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

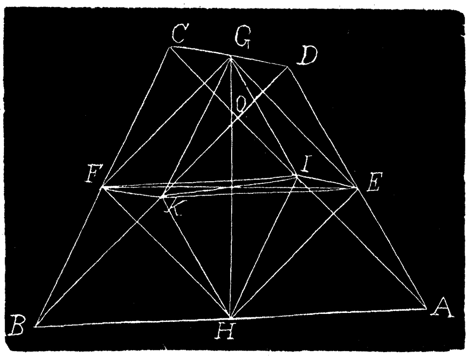
SOLUTIONS OF PROBLEMS.

43. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

The consecutive sides of a quadrilateral are a, b, c, d. Supposing its diagonals to be equal, find them and also the area of the quadrilateral.

II. Solution by A. H. BELL, Hillsboro, Illinois.

The solution as published simply demonstrates this theorem, $a^2+b^2+c^2+d^2=2x^2+4\overline{IK}^2$, with two unknowns, and is then solved for a particular case.



Let the sides of the quadrilateral AB, BC, CD, and AD, be a, b, c, and d; and the diagonals each $=2x$; $x+y$, $x-y$=the segments AO and OC; and BO and $OD=x+z$ and $x-z$. In the triangles AOB, BOC, and COD we have $(x+y)\cos A+(x+z)\cos B=a$, $\cos A=(4x^2+a^2-b^2)/(4ax)$, $\cos B=(4x^2+a^2-d^2)/(4ax)$, making